

Numerical study of the collapse of a bubble subjected to a lithotripter pulse

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Abstract.

The collapse of a bubble subjected to a lithotripter pulse is studied numerically. The goal is to record the pressure exerted along the stone, as a measure of potential stone damage. It is found that the pressure due to bubble collapse is much larger than that of the lithotripter pulse. Furthermore, the pressure greatly depends on the geometry of the problem (initial stand-off distance and bubble size) and on the properties of the pulse (amplitude and width).

Keywords: Bubble collapse, shockwave lithotripsy

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INTRODUCTION

In shockwave lithotripsy (SWL), shockwaves followed by a long expansion tail are focused near kidney stones in order to break them. The tensile part of the pulse causes pre-existing bubble nuclei to grow and subsequently collapse near a stone [1]. When a bubble collapses near a solid surface, a high-speed re-entrant jet directed towards the surface forms and penetrates the bubble. The jet eventually hits the distal side, thereby generating a large water-hammer pressure; for bubbles close enough to the stone, the direct impact of the jet onto the stone can cause significant damage [2]. The goal of the present study is to provide a quantitative understanding of the effect of bubble collapse in the stone comminution mechanism. To do so, the collapse of an air bubble subjected to a lithotripter pulse near a wall was studied numerically in [3].

NUMERICAL MODEL

Numerical simulations of compressible multiphase flows are particularly challenging. Interfaces separating fluids of very different densities can undergo large non-spherical deformations and interact with shockwaves; other complex physical phenomena such as diffusion, surface tension or phase change are important. Since the main features of the flow are interactions between various types of waves with interfaces, compressible multicomponent flows are considered. The fluid components are assumed immiscible, and phase change, surface tension and diffusive effects are neglected. The Euler equations, closed by a stiffened equation of state with appropriate advection equations, govern these flows. Because of shortcomings of current shock-capturing schemes, a new numerical method was developed in [4].

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PROBLEM SET-UP

A typical set-up for SWL is taken as the initial conditions: the initial bubble radius is $R_o = 50\mu\text{m}$, the pulse width is $\sigma = 6.6\text{mm}$, and the pressure amplitude is $P = 357\text{atm}$. The bubble is initially located at some distance, H , from a wall. Because the model equations do not include phase change, an exponentially decaying waveform is used, where the initial expansion matches the decay of a lithotripter pulse.

RESULTS AND DISCUSSION

Overall bubble behavior

Figure 1 shows Schlieren contours (norm of the density gradient), in order to visualize both interfaces and shockwaves. When the left-moving lithotripter pulse hits the bubble, an expansion wave is reflected, while a weak shockwave is transmitted into the bubble (frame 1). The pulse is then reflected off the wall on the left, as the bubble starts its aspherical collapse (frames 2-3). When the jet hits the distal side (frame 4), a large water-hammer pressure is generated, propagating radially outwards (frame 5). It is then reflected off the wall back onto the bubble (frame 6). After the water-hammer, the bubble becomes a vortex ring, which rebounds as it convects towards the wall (frames 7-8). The entire process occurs as the compressive part of the pulse is still entering the domain.

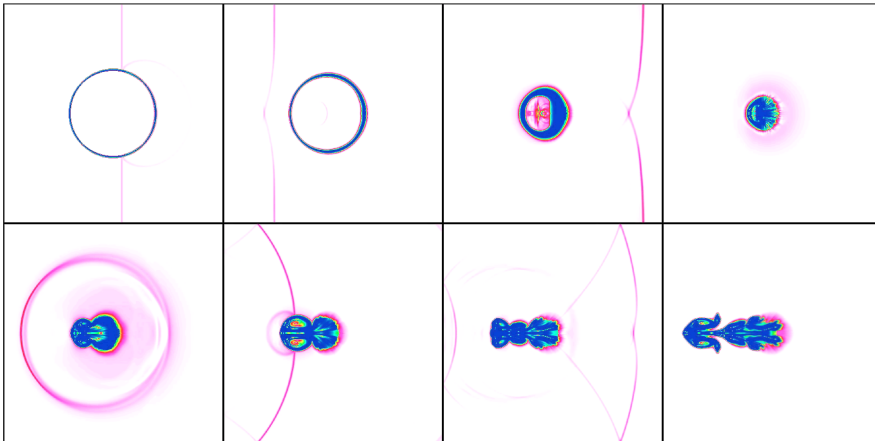


FIGURE 1. Idealized Schlieren contours of the collapse of an air bubble near a solid surface.

Geometrical effects

The effect of the initial stand-off distance from the wall is considered in the left frame of Figure 2. At the center of the domain, the wall pressure is much larger when the bubble is initially located close to the wall. At the top of the domain, the pressure is

almost constant, since the distance from the location of the water-hammer does not vary significantly, yet still much larger than the pressure due to the pulse. The wall pressure for small H/R_o is much larger than analytical results for underwater explosions; in this specific case, the bubble collides with the wall at a high speed.

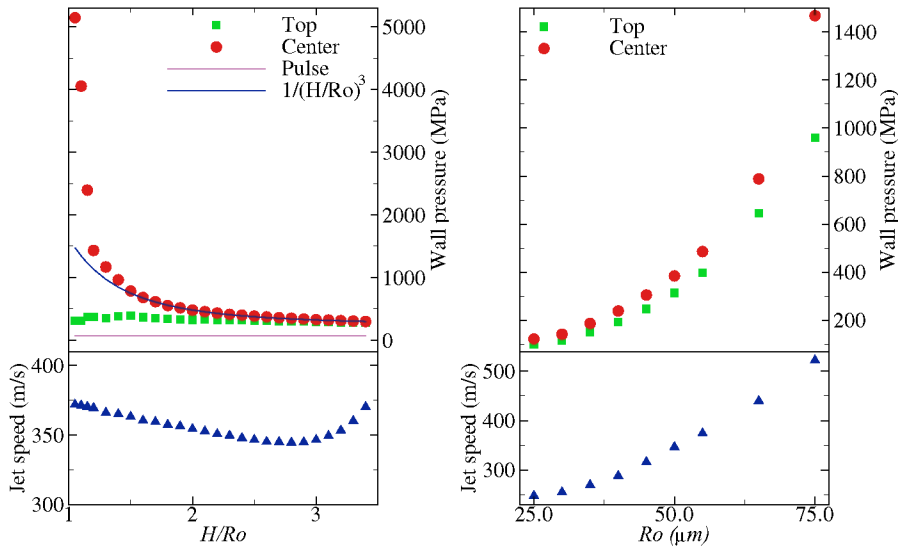


FIGURE 2. Maximum wall pressure and jet speed as a function of H/R_o and R_o .

The effect of the initial bubble radius is considered in the right frame of Figure 2. The jet speed increases with the bubble radius, so that the measured pressure increases accordingly. For a given pressure difference, a larger bubble will collapse to a smaller size (relative to its initial radius) than a smaller bubble.

Effects of the pulse properties

The effect of the pulse amplitude is considered in the left frame of Figure 3. The wall pressure increases linearly with the Mach number ¹. At the top of the domain, the full effect of the incoming lithotripter pulse is felt; the computed data matches the linearized theory well. The wall pressure due to the lithotripter pulse is smaller at the center than at the top of the domain, because the bubble “shields” the wall from the pulse. On the other hand, the wall pressure due to the bubble collapse is greater at the center than at the top of the domain.

Finally, the effect of the pulse width is considered in the right frame of Figure 3. The wall pressure increases with pulse width, since the bubble is subjected to a higher pressure for a longer time. As the width is increased, the pulse looks more and more

¹ The Mach number is related to the pressure ratio across the shock via normal shock relations.

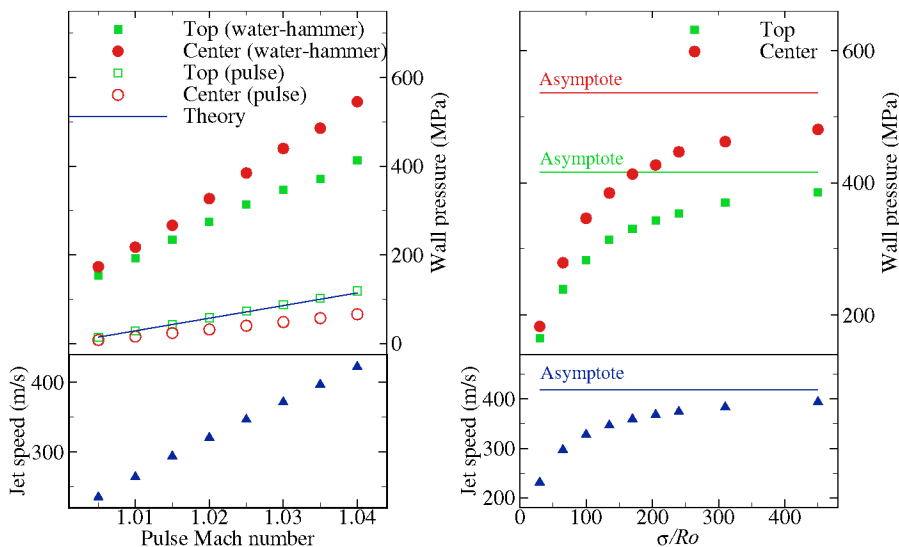


FIGURE 3. Maximum wall pressure and jet speed as a function of M and σ/R_o .

like a shockwave (with no expansion tail) to the bubble. This asymptotic value is shown using the straight lines, which are in reality a point located at $\sigma/R_o \rightarrow \infty$.

CONCLUSIONS

The presence of bubbles greatly amplifies the pressure measured at the surface of the stone in SWL. The pressure due to the bubble collapse (impact of the re-entrant jet onto the distal side) is much larger than that due to the initial pulse, even when the jet does not hit the stone. The geometry and properties of the pulse have important effects on the bubble dynamics. In future work, we will explore how the lithotripter design could be modified for optimal performance.

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